Availability Improvement of a Two Non identical Unit Hot Standby System using Substitute system

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Abstract: *In present paper two dissimilar units are considered, with feature of substitute system. The substitute system is connected on total failure of system, in order to have uninterrupted availability of the system services even when system is failed. Reliability characteristics are evaluated using Kolmogorov's forward equations. Repair times and failure times of each unit are assumed to be exponentially distributed.*

Key Words: *Linear first order differential equation, Mean Time to System Failure, Partial failure, Reliability, Steady State Availability.*

1. INTRODUCTION:

Many authors proposed many systems for different types of standby systems and evaluated mean time to system failure (MTSF), and availability models. [1] Evaluates mean time to system failure and availability modeling of three non-identical warm standby units. [4][5] Evaluates availability improvement of cold standby with substitute system. Availability is vital in competitive environment and present study is devoted to improve availability using substitute system on its total failure. In present paper a system is studied having two non-identical units hot standby repairable system. If repair of system can be completed in considerable time then repair will be continued and the system is brought back to the operative condition, otherwise some other substitute system is called, for continuation of operation, with guarantee of failure free operation to resume the desired operation. The substitute system is returned back when the original system starts working as good as new after repair.

2. NOTATIONS:

O = Operative unit.
H = Hot standby.
S = Substitute system.
F_r = Failed unit is under repair.
F_{wr} = Failed unit is waiting for repair.
a = Constant failure rate of 1st unit.
b = Constant failure rate of 2nd unit.
c = Constant rate of connecting substitute system.
d = Constant repair rate of a 1st unit.
f = Constant repair rate of a 2nd unit.
g = Constant repair rate of failed system.

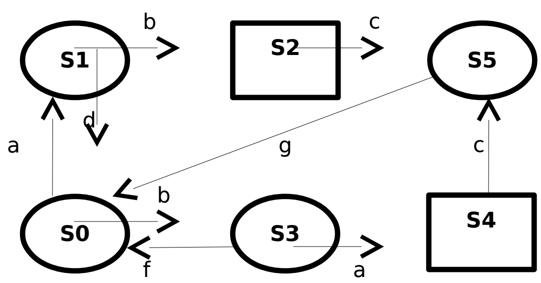

Figure 1

2.1. States of the system

$S_0(O,H)$, $S_1(F_r ,O)$, $S_2(F_r, F_{wr})$, $S_3(O, F_r)$, $S_4(F_{wr},F_r)$, $S_5(S)$

3. MEAN TIME TO SYSTEM FAILURE (MTSF):

The mean time to system failure (MTSF) for the proposed system is evaluated using the linear first order differential equations. Let $P_i(t)$ is the probability that the system at time t, $(t \geq 0)$ is in state S_i. Let P (t) denote the probability row vector at time t, the initial conditions for this problem are:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)]$$
$$= [1, 0, 0, 0, 0, 0] \quad \ldots (1)$$

By employing the method of linear first order differenti al equations, we obtain the following differenti al equations :

$$\frac{dP_0}{dt} = -(a+b)P_0 + dP_1 + fP_3 + gP_5$$
$$\frac{dP_1}{dt} = -(b+d)P_1 + aP_0$$
$$\frac{dP_2}{dt} = -cP_2 + bP_1$$

$$\frac{dP_3}{dt} = -(a+f)P_3 + bP_0$$
$$\frac{dP_4}{dt} = -cP_4 + aP_3$$
$$\frac{dP_5}{dt} = -gP_5 + cP_2 + cP_4$$

This can be written in the matrix form as:
P *= Q P, where

$$Q = \begin{bmatrix} -(a+b) & d & 0 & f & 0 & g \\ a & -(b+d) & 0 & 0 & 0 & 0 \\ 0 & b & -c & 0 & 0 & 0 \\ b & 0 & 0 & -(a+f) & 0 & 0 \\ 0 & 0 & 0 & a & -c & 0 \\ 0 & 0 & c & 0 & c & -g \end{bmatrix}$$

To calculate the MTSF, we take the transpose of the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called (A). The expected time to reach an absorbing state is calculated from

$$E[T_{P(0) \to P(absorbing)}] = P(0)(-A^{-1})\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \text{where} \qquad ...(2)$$

$$A = \begin{bmatrix} -(a+b) & a & b & 0 \\ d & -(b+d) & 0 & 0 \\ f & 0 & -(a+f) & 0 \\ g & 0 & 0 & -g \end{bmatrix}$$

We obtain the following expression for MTSF on solving equation (2).

$$MTSF = \frac{b^2 + (d+a+f)\,b + (a+f)\,d + a^2 + f\,a}{a\,d + a\,b^2 + (a^2 + f\,a)\,b}$$

4. AVAILABILITY ANALYSIS OF THE SYSTEM:

The initial condition for this problem is same as for the reliability case i.e.

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1, 0, 0, 0, 0, 0]$

This can be written in the matrix form as:
P *= Q P, where

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \\ P_5^* \end{bmatrix} = \begin{bmatrix} -(a+b) & d & 0 & f & 0 & g \\ a & -(b+d) & 0 & 0 & 0 & 0 \\ 0 & b & -c & 0 & 0 & 0 \\ b & 0 & 0 & -(a+f) & 0 & 0 \\ 0 & 0 & 0 & a & -c & 0 \\ 0 & 0 & c & 0 & c & -g \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

Let t be the time to failure of the system. Then steady-state availability is given by

$A_t(\infty) = 1-[P_2(\infty) + P_4(\infty)]$

In the steady-state situation, the derivatives of the state probabilities become zero. That is

$QP(\infty) = 0$

Then above matrix becomes

$$\begin{bmatrix} -(a+b) & d & 0 & f & 0 & g \\ a & -(b+d) & 0 & 0 & 0 & 0 \\ 0 & b & -c & 0 & 0 & 0 \\ b & 0 & 0 & -(a+f) & 0 & 0 \\ 0 & 0 & 0 & a & -c & 0 \\ 0 & 0 & c & 0 & c & -g \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

On substituting the normalizing condition $\Sigma_{i=0}^{5} P_i(\infty) = 1$, in any one of the redundant rows of above matrix and on solving, the solution provides the steady-state probabilities $P_0(\infty)$, $P_1(\infty)$, …, $P_5(\infty)$. Expression for steady-state availability is thus

$$A(\infty) = \frac{N_1}{D_1}, \text{where}$$

$$N_1 = g[(c\,d + bc + ba + a\,c)\,f + (bc + ba + a\,c)\,d + ba\,c + ba^2 + b^2(c + a) + a^2\,c] + a\,b\,c\,f + a\,b\,c\,d + a\,b^2\,c + a^2\,b\,c - g[a\,b\,f + a^2\,b + a\,b\,d + a\,b^2]$$

$$D_1 = g[(c\,d + bc + ba + a\,c)\,f + (bc + ba + a\,c)\,d + ba\,c + ba^2 + b^2(c + a) + a^2\,c] + a\,b\,c\,f + a\,b\,c\,d + a\,b^2\,c + a^2\,b\,c$$

5. PARTICULAR CASE:

When Substitute system is not connected the availability of the system is obtained as:

$$A_t(\infty) = 1 - [P_2(\infty) + P_4(\infty)]$$

$$A(\infty) = 1 - \frac{N_2}{D_2}, \textit{where}$$

$$N_2 = [(d + a)\,f^2 + (b + a)\,df + a^2\,f + a\,b\,d]a\,b\,f + a\,b\,d[d^2f + (b + a)\,df + a\,b\,f + b\,d^2 + b^2\,d]$$

$$D_2 = [(d^2 + (b + a)\,d + a\,b)\,f + b\,d^2 + b^2\,d][(d + a)\,f^2 + (b + a)\,df + a^2\,f + a\,b\,d]$$

6. MATERIAL AND METHOD:

In this study system is analysed by making use of Kolmogorov's forward equations method. Numerically measures system effectiveness such as Mean time to system failure and Availability and plotted graph to show effect of additional feature, substitute system on availability.

7. RESULTS:

To study the effect of proposed system on availability, numerically and graphically the proposed system is compared with availability of the system of special case that does not have substitute system. For sake of comparison the value of parameters are fixed for consistency:

$0.1 \le a \le 0.7$, $b = 0.2$, $c = 0.1$, $d = 0.3$, $f = 0.3$, $g = 0.5$

Table 1: Relation between failure rate of first unit and the availability (with and without substitute system)

Failure rate of first unit (a)	Availability with substitute system	Availability without substitute system
0.1	0.963144963	0.818181818181818
0.2	0.942742628	0.724137931034483
0.3	0.927916121	0.666666666666667
0.4	0.915745129	0.627906976744186
0.5	0.905109489	0.6
0.6	0.895483389	0.578947368421053

0.7	0.886584576	0.5625

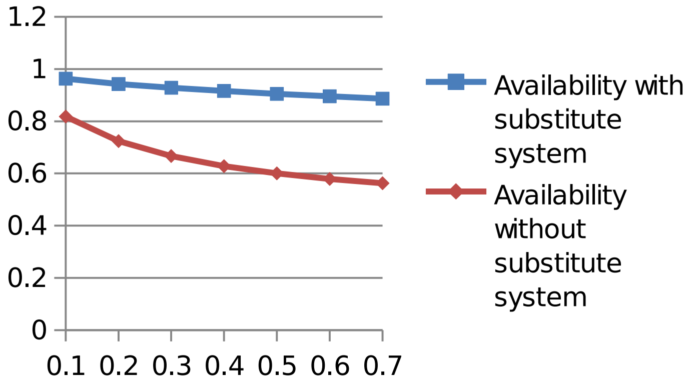

Figure 2

8. DISCUSSIONS:

By comparing the availability with respect to failure rate of first unit for proposed system and particular case numerically and graphically, it is observed that increase of failure rate (a) at constant b = 0.2, c = 0.1, d = 0.3, f = 0.3, g = 0.5 decreased the availability for both the systems with and without substitute system.

Also graph shows that availability of the system with substitute (proposed system) is greater than the system without substitute system (particular case) with respect to failure rate of system.

9. CONCLUSION:

It is concluded that: the system with substitute system improves availability and provides better availability than the systems without substitute systems.

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