Fuzzy Inventory Model for Weibull Deteriorating Items with Exponential Demand and Shortages under Fully Backlogged Conditions

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Abstract: *The aim of this paper is to develop a fuzzy inventory model for deteriorating items which follow the Weibull deterioration and exponential demand and shortages under fully backlogged. The deterioration cost, holding cost and shortages cost are taken as hexagonal fuzzy members. Singed distance method is used to defuzzify the total cost function. The numerical are given in order to show the applicability of the proposed model.*

Keywords: *Exponential demand, Weibull deterioration, Shortages , Hexagonal fuzzy numbers, Defuzzification, Signed distance method.*

1. INTRODUCTION:

Inventory management is one of the most challenging in the organization and recently it has been taken seriously managers. In 1915, the first inventory model was developed by F.Harris [1] , In the business world deterioration is an important key factor. In general, deterioration is define as the damage, spoilage, dryness, vaporization etc. , that results in decrease of usefulness of the original one. Inventory problems for deteriorating items have been widely studied [3],[4],[5],[6],[8],[10]. In our daily life, most of the physical goods like medicine, alcohols, volatile liquids, blood banks, fresh products, flowers, food grains, fruits, vegetables, seafood undergoes decay or deterioration over time. Then Emmons (1968) proposed this type of model with variable deterioration, which follows two-parameter Weibull distribution. [5] who developed an EOQ model of deteriorating items. In the business world demand of the item is a most important key factor. The demand of an item depends on various key factors like, selling price, showroom at the market place, stock market demand etc. [11] gupta and vrat were first develop models for stock dependent consumption rate. More about related papers are also [2], [6], [8], [10].

In many inventory models uncertainty is due to fuzziness and fuzziness is the closed possible approach to reality. First time in 1965, L. A. Zadeh [13] introduced the concept of fuzzy sets. The theory of fuzzy sets attracted by many researchers. S.K.Indrajitsingha, P.N. Samanta & U.K.Mishra [6] developed an Economic production quantity model with time dependent demand under fuzzy environment and also developed [7] , Fuzzy inventory model with shortages under fully backlogged using signed distance method. More fuzzy related papers are also [3], [4], [8], [9].

In this paper, an attempt is made to formulate the mathematical fuzzy model for inventory system with exponential demand. Demand rate is considered to be an increasing function of time. Shortages are allowed and fully backlogged. Deteriorating items are following the two parameter Weibull distribution.

The rest of paper organized as follows: Definitions and preliminaries are given in section 2, Notations and assumptions which are used throughout this article, are descried in section 3, and followed by mathematical scrip model in section 4 and mathematical fuzzy model in section 5. Section 6, to illustrate the numerical examples and Sensitivity analysis is provided for variation of various parameters to illustrate the proposed model. Finally, conclusions are draw and the future researches are pointed out in section 8.

2. DEFINITIONS AND PRELIMINARIES:

In order to establish the model we require the following definitions:

Definition 2.1 (Fuzzy Set) : A fuzzy set $\tilde{A}$ in a universe of discourse x is defined as the following set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \epsilon X\}$. Here $\mu_{\tilde{A}}: X \to [0,1]$ is a mapping called the membership value of $x \epsilon X$ in a fuzzy set $\tilde{A}$.

Definition 2.2 (Convex Fuzzy Set) A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \epsilon X\} \subseteq X$ is called a convex fuzzy set if for any $x_1, x_2 \in X$ (universe)

$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)x) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \quad for\ all\ \lambda \epsilon [0,1]$$

Definition 2.3 (Fuzzy number) Fuzzy number is a fuzzy set which both convex and normal.

Definition 2.4 Let $a, b \epsilon R$ such that $a < b$. Then for $0 \leq \alpha \leq 1$, the fuzzy set $[a_\alpha, b_\alpha]$ is called a fuzzy interval if its membership function is

$$\mu_{[a_\alpha, b_\alpha]} = \begin{cases} \alpha, & a \leq x \leq b \\ 0, & otherwise \end{cases}$$

Definition 2.5 Let $a, b, c \epsilon R$ such that $< b < c$. Then the fuzzy number

$\tilde{A} = (a, b, c)$ is called a triangular fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} , & a \le x \le b \\ \frac{c-x}{c-b} , & b \le x \le c \\ 0 , & otherwise \end{cases}$$

Definition 2.6 A hexagonal fuzzy number $\tilde{A} = (a, b, c, d, e, f)$ is represented with membership function $\mu_{\tilde{A}}$ as

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \frac{1}{2}\left(\frac{x-a}{b-a}\right), & a \le x \le b \\ L_2(x) = \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-d}\right), & b \le x \le c \\ 1 , & c \le x \le d \\ R_1(x) = 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right), & d \le x \le e \\ R_2(x) = \frac{1}{2}\left(\frac{f-x}{f-c}\right) , & e \le x \le f \\ 0 , & otherwise \end{cases}$$

The α-cut of $\tilde{A} = (a, b, c, d, e, f)$, $0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where

$$A_{L_1}(\alpha) = a + (b - a)\alpha = L_1^{-1}(\alpha)$$
$$A_{L_2}(\alpha) = b + (c - b)\alpha = L_2^{-1}(\alpha)$$
$$A_{R_1}(\alpha) = e + (e - d)\alpha = R_1^{-1}(\alpha)$$
$$A_{R_2}(\alpha) = f + (f - e)\alpha = R_2^{-1}(\alpha)$$

Therefore

$$L^{-1}(\alpha) = \frac{L_1^{-1}(\alpha) + L_2^{-1}(\alpha)}{2} = \frac{a+b+(c-a)\alpha}{2}$$
$$R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha) + R_2^{-1}(\alpha)}{2} = \frac{e + f + (d - f)\alpha}{2}$$

Definition 2.7 If $\tilde{A} = (a, b, c, d, e, f)$ is a hexagonal fuzzy number then the signed distance method of $\tilde{A}$ is defined as

$$d(\tilde{A}, \tilde{0}) = \int_0^1 d([A_L(\alpha), A_R(\alpha)], \tilde{0}) = \frac{a+2b+c+d+2e+f}{8}$$

Definition 2.8 Suppose $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers and $a_i, b_i \epsilon R$, $i = 1,2,3,4,5,6$ then the arithmetical operations are define as

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$$
$$\tilde{A} \ominus \tilde{B} = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$$
$$\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5, a_6 b_6)$$
$$-\tilde{A} = (-a_6, -a_5, -a_4, -a_3, -a_2, -a_1)$$
$$\frac{1}{\tilde{A}} = \tilde{A}^{-1} = \left(\frac{1}{a_6}, \frac{1}{a_5}, \frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right)$$
$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_6}, \frac{a_2}{b_5}, \frac{a_3}{b_4}, \frac{a_4}{b_3}, \frac{a_5}{b_2}, \frac{a_6}{b_1}\right)$$
$$\alpha \otimes \tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha a_6) , & \alpha \ge 0 \\ (\alpha a_6, \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1) , & \alpha < 0 \end{cases}$$

3. ASSUMPTIONS AND NOTATIONS:

In developing the model we assume:

(i) Consider only single items
(ii) Lead time is zero.
(iii) Time horizon is infinite.
(iv) Replenishment rate is infinite.
(v) Shortages are allowed and fully backlogged.
(vi) There is no repair of the deteriorated items occurring during the cycle.
(vii) Demand consider exponential.
(viii) Deterioration follow two parameters Weibull distribution deterioration rate.

Notations:

(i) $I(t) =$ inventory level at time $t, t \ge 0$.
(ii) $D(t) = ae^{bt}$ is demand rate at time *t*.
(iii) $\theta(t) = \mu\lambda t^{\mu-1}$, Weibull distribution deterioration rate. Where $0 < \lambda \le 1$ is called the scale parameter, $\mu > 0$ is the shape parameter.

(iv) $h=$ holding cost per unit item.
(v) $s=$ shortage cost per unit item.
(vi) $d=$ deterioration cost per unit item.
(vii) $Q=$ inventory level at time $t=0$.
(viii) $Z[A,C]=$ total average cost of the system per unit time.
(ix) $\tilde{H}$ =fuzzy holding cost.
(x) $\tilde{S}$ =fuzzy shortage cost.
(xi) $\tilde{D}$ =fuzzy deterioration cost.
(xii) $\tilde{Z}[A,C]=$ total fuzzy cost of the system per unit time.
(xiii) $\tilde{Z}[A,C,S]=$ defuzzified value of $\tilde{Z}[A,C]$ by applying signed distance method.
(xiv) $T=$ duration of the cycle.

4. MATHEMATICAL FORMULATION:

We assume that inventory level at time $t=0$ is Q. The inventory level gradually decreases to meet the demand of the customers and deterioration and ultimately falls to zero at $t=t_1$. Shortage occurs in the time interval $[t_1,T]$ and which are fully backlogged. Therefore the inventory level form can be expressed by the following equation.

The inventory level is governed by the following differential equations:

$$\frac{dI(t)}{dt}+\lambda\mu t^{\mu-1}I(t)=-ae^{bt}, 0\le t\le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt}=-ae^{bt}, t_1\le t\le T \qquad (2)$$

Now solving (1) with boundary condition $I(t_1)=0$

$$I(t)=ae^{-\lambda t^{\mu}}\left[(t_1-t)+\frac{\lambda}{\mu+1}\left(t_1^{\mu+1}-t^{\mu+1}\right)+\frac{b}{2}(t_1^2-t^2)+\frac{b^2}{6}(t_1^3-t^3)+\frac{b\lambda}{\mu+2}\left(t_1^{\mu+2}-t^{\mu+2}\right)\right], \quad 0\le t \le t_1 \qquad (3)$$

Neglecting the square and higher order of λ.

Using initial condition $I(0)=Q$ we have

$$Q = a\left[t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right] \qquad (4)$$

Now solving (2) and using boundary condition we have $I(t)=\frac{a}{b}\left(e^{bt_1}-e^{bt}\right), t_1\le t\le T \qquad (5)$

Holding cost per cycle

$$\int_0^{t_1}I(t)dt=h\int_0^{t_1}ae^{-\lambda t^{\mu}}\left[(t_1-t)+\frac{\lambda}{\mu+1}\left(t_1^{\mu+1}-t^{\mu+1}\right)+\frac{b}{2}(t_1^2-t^2)+\frac{b^2}{6}(t_1^3-t^3)+\frac{b\lambda}{\mu+2}\left(t_1^{\mu+2}-t^{\mu+2}\right)\right]dt$$

$$=h\left[\frac{Q}{a}t_1-\frac{t_1^2}{2}-\frac{b}{6}t_1^3-\frac{b^2}{24}t_1^4-\frac{Q\lambda}{a}\frac{t_1^{\mu+1}}{\mu+1}+\frac{\lambda\mu}{\mu+1}\frac{t_1^{\mu+2}}{\mu+2}+\frac{\lambda\mu b}{2(\mu+1)}.\frac{t_1^{\mu+3}}{\mu+3}+\frac{b^2}{6}\frac{t_1^{\mu+4}}{\mu+4}\right] \qquad (6)$$

Deterioration cost per cycle

$$d\int_0^{t_1}\lambda\mu t^{\mu-1}I(t)dt$$

$$=d\int_0^{t_1}\lambda\mu t^{\mu-1}ae^{-\lambda t^{\mu}}\left[(t_1-t)+\frac{\lambda}{\mu+1}\left(t_1^{\mu+1}-t^{\mu+1}\right)+\frac{b}{2}(t_1^2-t^2)+\frac{b^2}{6}(t_1^3-t^3)+\frac{b\lambda}{\mu+2}\left(t_1^{\mu+2}-t^{\mu+2}\right)\right]dt$$

$$=d\lambda\mu a\left[\frac{Q}{a\mu}t_1^{\mu}-\frac{Q\lambda}{2a\mu}t_1^{2\mu}-\frac{t_1^{\mu+1}}{\mu+1}-\frac{b}{2}\frac{t_1^{\mu+2}}{\mu+2}-\frac{b^2}{6}\frac{t_1^{\mu+3}}{\mu+3}+\frac{\mu\lambda}{\mu+1}\frac{t_1^{2\mu+1}}{2\mu+1}+\frac{b\lambda}{2}\frac{t_1^{2\mu+2}}{2\mu+2}-\frac{b\lambda}{2\mu+3}\left(\frac{1}{\mu+2}-\frac{b}{6}\right)t_1^{2\mu+3}\right] \qquad (7)$$

Shortages cost per cycle

$$s\int_{t_1}^{T} I(t)dt = s\int_{t_1}^{T}\frac{a}{b}\left(e^{bt_1}-e^{bt}\right)dt$$

$$=\frac{sa}{b}\left\{e^{bt_1}(T-t_1)-\frac{1}{b}\left(e^{bT}-e^{bt_1}\right)\right\} \qquad (8)$$

The total average cost per unit time of the model is

$$Z[A,C](T,t_1)=\frac{1}{T}\{Holding\ cost + Deteriorating\ cost + Shortages\ cost\}$$

$$=\frac{1}{T}\Bigg[h\left\{\frac{Q}{a}t_1-\frac{t_1^2}{2}-\frac{b}{6}t_1^3-\frac{b^2}{24}t_1^4-\frac{Q\lambda}{a}\frac{t_1^{\mu+1}}{\mu+1}+\frac{\lambda\mu}{\mu+1}\frac{t_1^{\mu+2}}{\mu+2}+\frac{\lambda\mu b}{2(\mu+1)}.\frac{t_1^{\mu+3}}{\mu+3}+\frac{b^2}{6}\frac{t_1^{\mu+4}}{\mu+4}\right\}+d\lambda\mu a\left\{\frac{Q}{a\mu}t_1^{\mu}-\frac{Q\lambda}{2a\mu}t_1^{2\mu}-\frac{t_1^{\mu+1}}{\mu+1}-\frac{b}{2}\frac{t_1^{\mu+2}}{\mu+2}-\frac{b^2}{6}\frac{t_1^{\mu+3}}{\mu+3}+\frac{\mu\lambda}{\mu+1}\frac{t_1^{2\mu+1}}{2\mu+1}+\frac{b\lambda}{2}\frac{t_1^{2\mu+2}}{2\mu+2}-\frac{b\lambda}{2\mu+3}\left(\frac{1}{\mu+2}-\frac{b}{6}\right)t_1^{2\mu+3}\right\}+\frac{sa}{b}\left\{e^{bt_1}(T-t_1)-\frac{1}{b}\left(e^{bT}-e^{bt_1}\right)\right\}\Bigg] \qquad (9)$$

$$=\frac{1}{T}\Bigg[h\Bigg\{\left(t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right)t_1-\frac{t_1^2}{2}-\frac{b}{6}t_1^3-\frac{b^2}{24}t_1^4-\lambda\left(t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right)\frac{t_1^{\mu+1}}{\mu+1}+\frac{\lambda\mu}{\mu+1}\frac{t_1^{\mu+2}}{\mu+2}+\frac{\lambda\mu b}{2(\mu+1)}.\frac{t_1^{\mu+3}}{\mu+3}+\frac{b^2}{6}\frac{t_1^{\mu+4}}{\mu+4}\Bigg\}+d\lambda\mu a\Bigg\{\frac{1}{\mu}\left(t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right)t_1^{\mu}-\frac{\lambda}{2\mu}\left(t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right)t_1^{2\mu}-\frac{t_1^{\mu+1}}{\mu+1}-\frac{b}{2}\frac{t_1^{\mu+2}}{\mu+2}-\frac{b^2}{6}\frac{t_1^{\mu+3}}{\mu+3}+\frac{\mu\lambda}{\mu+1}\frac{t_1^{2\mu+1}}{2\mu+1}+\frac{b\lambda}{2}\frac{t_1^{2\mu+2}}{2\mu+2}-\frac{b\lambda}{2\mu+3}\left(\frac{1}{\mu+2}-\frac{b}{6}\right)t_1^{2\mu+3}\Bigg\}+\frac{sa}{b}\left\{e^{bt_1}(T-t_1)-\frac{1}{b}\left(e^{bT}-e^{bt_1}\right)\right\}\Bigg] \qquad (10)$$

The necessary condition for $Z[A,C](T,t_1)$ to be minimum is that

$$\frac{\partial Z[A,C](T,t_1)}{\partial t_1}=0 \ \text{ and } \ \frac{\partial Z[A,C](T,t_1)}{\partial T}=0$$

Solving these equations we find the optimum values of t_1 and T say t_1^* and T^* for which cost is minimum and the sufficient condition is

$$\frac{\partial^2 Z[A,C](T,t_1)}{\partial t_1^2}.\frac{\partial^2 Z[A,C](T,t_1)}{\partial T^2}-\frac{\partial^2 Z[A,C](T,t_1)}{\partial T\partial t_1}>0$$

5. FUZZY MODEL:

Now consider this inventory model in fuzzy environment due to uncertainty in parameters let us assume that the parameters $\tilde{H},\tilde{S},\tilde{D}$.

Consider the non-negative hexagonal fuzzy numbers are following

$\tilde{H}=(h_1,h_2,h_3,h_4,h_5,h_6)$
$\tilde{S}=(s_1,s_2,s_3,s_4,s_5,s_6)$
$\tilde{D}=(d_1,d_2,d_3,d_4,d_5,d_6)$

Total cost of the system per unit time in fuzzy sense is given by

$$\tilde{Z}[A,C](T,t_1)=\left(\eta\otimes\tilde{H}\right)\oplus\left(\kappa\otimes\tilde{D}\right)\oplus\left(\tau\otimes\tilde{S}\right)$$

Where

$$\eta=\frac{1}{T}\Bigg[t_1\left\{t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right\}-\frac{t_1^2}{2}-\frac{b}{6}t_1^3-\frac{b^2}{24}t_1^4-\frac{\lambda t_1^{\mu+1}}{\mu+1}\left\{t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right\}+\frac{\lambda\mu}{\mu+1}\frac{t_1^{\mu+2}}{\mu+2}+\frac{\lambda\mu b}{2(\mu+1)}.\frac{t_1^{\mu+3}}{\mu+3}+\frac{b^2}{6}\frac{t_1^{\mu+4}}{\mu+4}\Bigg]$$

$$\kappa=\frac{d\lambda\mu a}{T}\Bigg[\frac{t_1^{\mu}}{\mu}\left\{t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right\}-\frac{t_1^{2\mu}\lambda}{2\mu}\left\{t_1+\frac{\lambda}{\mu+1}t_1^{\mu+1}+\frac{b}{2}t_1^2+\frac{b^2}{6}t_1^3+\frac{b\lambda}{\mu+2}t_1^{\mu+2}\right\}-\frac{t_1^{\mu+1}}{\mu+1}-\frac{b}{2}\frac{t_1^{\mu+2}}{\mu+2}-\frac{b^2}{6}\frac{t_1^{\mu+3}}{\mu+3}+\frac{\mu\lambda}{\mu+1}\frac{t_1^{2\mu+1}}{2\mu+1}+\frac{b\lambda}{2}\frac{t_1^{2\mu+2}}{2\mu+2}-\frac{b\lambda}{2\mu+3}\left(\frac{1}{\mu+2}-\frac{b}{6}\right)t_1^{2\mu+3}\Bigg]$$

$$\tau=\frac{sa}{b\,T}\left\{e^{bt_1}(T-t_1)-\frac{1}{b}\left(e^{bT}-e^{bt_1}\right)\right\}$$

$$\tilde{Z}[A,C](T,t_1)=\begin{pmatrix}\tilde{Z}[A,C,1](T,t_1),\tilde{Z}[A,C,2](T,t_1),\tilde{Z}[A,C,3](T,t_1),\tilde{Z}[A,C,4](T,t_1),\tilde{Z}[A,C,5](T,t_1),\\ \tilde{Z}[A,C,6](T,t_1)\end{pmatrix}$$

Where

$\tilde{Z}[A,C,i](T,t_1)=\eta h_i+\kappa d_i+\tau s_i, \qquad i=1,2,3,4,5,6$

Now by defuzzifying the fuzzy total average cost $\tilde{Z}[A,C](T,t_1)$ by signed distance method, we get

$$\tilde{Z}[A,C](T,t_1)=\frac{1}{8}[\tilde{Z}[A,C,1](T,t_1)+2\tilde{Z}[A,C,2](T,t_1)+\tilde{Z}[A,C,3](T,t_1)+\tilde{Z}[A,C,4](T,t_1)+2\tilde{Z}[A,C,5](T,t_1)+\tilde{Z}[A,C,6](T,t_1)]$$

$$\tilde{Z}[A,C,S](T,t_1)=\frac{\eta}{8}[h_1+2h_2+h_3+h_4+2h_5+h_6]+\frac{\kappa}{8}[d_1+2d_2+d_3+d_4+2d_5+d_6]+\frac{\tau}{8}[s_1+2s_2+s_3+s_4+2s_5+s_6]$$

The necessary condition for minimizing the total average cost is

$$\frac{\partial\tilde{Z}[A,C,S](T,t_1)}{\partial t_1}=0 \text{ and } \frac{\partial\tilde{Z}[A,C,S](T,t_1)}{\partial T}=0$$

Solving these equations we find the optimum values of t_1 and T say t_1^* and T^* for which cost is minimum and the sufficient condition is

$$\frac{\partial^2\tilde{Z}[A,C,S](T,t_1)}{\partial t_1^2}.\frac{\partial^2\tilde{Z}[A,C,S](T,t_1)}{\partial T^2}-\frac{\partial^2\tilde{Z}[A,C,S](T,t_1)}{\partial T\partial t_1}>0$$

$$\frac{\partial\tilde{Z}[A,C,S](T,t_1)}{\partial t_1}=\frac{1}{8}[h_1+2h_2+h_3+h_4+2h_5+h_6]\frac{\partial\eta}{\partial t_1}+\frac{1}{8}[d_1+2d_2+d_3+d_4+2d_5+d_6]\frac{\partial\kappa}{\partial t_1}+\frac{1}{8}[s_1+2s_2+s_3+s_4+2s_5+s_6]\frac{\partial\tau}{\partial t_1}$$

$$\frac{\partial\tilde{Z}[A,C,S](T,t_1)}{\partial T}=\frac{1}{8}[h_1+2h_2+h_3+h_4+2h_5+h_6]\frac{\partial\eta}{\partial T}+\frac{1}{8}[d_1+2d_2+d_3+d_4+2d_5+d_6]\frac{\partial\kappa}{\partial T}+\frac{1}{8}[s_1+2s_2+s_3+s_4+2s_5+s_6]\frac{\partial\tau}{\partial T}$$

Thus minimum value of the total cost is

$$\tilde{Z}[A,C,S](T,t_1)=\frac{1}{8}[\eta'h_1+\kappa'd_1+\tau's_1]+\frac{1}{4}[\eta'h_2+\kappa'd_2+\tau's_2]+\frac{1}{8}[\eta'h_3+\kappa'd_3+\tau's_3]+\frac{1}{8}[\eta'h_4+\kappa'd_4+\tau's_4]+\frac{1}{4}[\eta'h_5+\kappa'd_5+\tau's_5]+\frac{1}{8}[\eta'h_6+\kappa'd_6+\tau's_6]$$

6. NUMERICAL EXAMPLES:

To illustrate the above model the following input data are considered.

1. The parametric values in proper units.
   Suppose $\tilde{H}=(20,30,40,40,50,60)$
   $\tilde{S}=(10,20,30,30,40,50)$
   $\tilde{D}=(40,50,60,60,70,80)$
   $\lambda=0.001,\mu=2,a=500,b=5,t_1=0.9166,T=1$
   Hence the total average optimal cost defuzzified in signed distance method is $\tilde{Z}[A,C,S](T,t_1)=6147.60$ and $Q=3113.41$ (in proper units)

2. The parametric values in proper units.
   Suppose $\tilde{H}=(20,30,40,40,50,60)$
   $\tilde{S}=(10,20,30,30,40,50)$
   $\tilde{D}=(40,50,60,60,70,80)$
   $\lambda=0.001,\mu=3,a=600,b=5,t_1=0.9166,T=1$
   Hence the total average optimal cost defuzzified in signed distance method is $\tilde{Z}[A,C,S](T,t_1)=7313.91$ and $Q=3735.90$ (in proper units)

7. SENSITIVITY ANALYSIS:

Sensitivity analysis are performed for different values of λ,μ,a,b. It is observed that if a and b are fixed and for different values of λ is fixed and μ increases, inventory cost and Q both are decreases also when μ is fixed and λ increases, inventory cost and Q both are increases (table-1). And if λ and μ are fixed and for different values of a as b increases, inventory cost and Q both are increases (table-2). Observation table is given below.

Table-1

λ	**μ**	**Inventory Cost(Z)**	**Q**
0.001	2	758.53	3736.09
	3	755.55	3735.90
	4	753.52	3735.78
0.005	2	719.59	3738.83
	3	780.25	3737.88
	4	772.65	3737.28
0.010	2	832.89	3742.24
	3	811.11	3740.35
	4	796.54	3739.15

Table-2

b	**a**	**Inventory Cost(Z)**	**Q**
5	600	755.53	3735.90
	700	874.42	4358.55
	800	993.29	4981.21
7	600	4779.27	6088.36
	700	5563.86	7103.09
	800	6348.45	8117.81
9	600	31379.97	9056.89
	700	36591.76	10566.37
	800	41803.55	12075.85

8. CONCLUTION:

This paper present a fuzzy inventory model for two parameters Weibull deteriorating items and shortages under fully backlogged and exponential demand. The demand, holding and shortage cost are represented by hexagonal fuzzy numbers. To evaluate the total fuzzy cost, signed distance method is used to defuzzification. Numerical example is given to validate the proposed. Mathematical model which has been developed for determining the optimal total inventory cost.

In the future study, it is hoped to further incorporate the proposed model into more realistic assumption, such as probabilistic demand, introduce shortages and partial backlogged, generalize the model under two-level credit period strategy etc.

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