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Relations of Domination Parameter in Intuitionistic Fuzzy Graph

V. Pachamuthu.
Assistant Professor, Department of Mathematics, Vivekanandha College of Arts and Sciences for Women, Tiruchengode, Tamilnadu, India.
Email: pachamuthu1729@gmail.com

Abstract: *In this paper we introduced some relationship among the domination parameter in intuitionistic fuzzy graph. Particularly the parameters are domination number, non-split domination number, split domination number and complementary nil domination number. Some properties of dominating set, split and non-split domination in IFG are discussed.*

Key Words: *Minimum domination number, Maximum domination number, Minimum non-split domination number, Maximum non-split domination number, Minimum split domination number, Maximum split domination number, Minimum complementary nil domination number, Maximum complementary nil domination number.*

1. INTRODUCTION:

The concept of Intuitionistic Fuzzy Set(IFS) as a generalization of fuzzy set was introduced by K.T.Atanassov. He included a new component in the definition of fuzzy set which determines the degree of non-membership. IFS give a degree of membership, a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one and the degree of hesitation called indeterminancy (and is defined as one minus the sum of membership and non-membership degree respectively). K.T.Atanassov[1] introduced the concepts of intuitionistic fuzzy relation and intuitionistic fuzzy graphs using five types of Cartesian products. R.Parvathi and M.G.Karunambigai[9] defined intuitionistic fuzzy graph. A.NagoorGani and S.ShajithaBegum[6] defined order, degree and size in intuitionistic fuzzy graph. R.Parvathi and G.Tamizhendhi[11] introduced the concept of domination in intuitionistic fuzzy graph. R.Jahir Hussain and S.Yahaya Mohamed[5] gave the definition of complimentary nil domination in intuitionistic fuzzy graph. A.NagoorGani and S.Anupriya[7,8] introduced the concept of non-split domination and split domination in intuitionistic fuzzy graph.

2. PRELIMINARIES:

In this section, some basic definitions relating to IFGs are given. Also the definition of domination number, non-split domination number, split domination number and complementary nil domination number in IFG are discussed.

Definition 2.1. An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$, where (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degree of membership and non - membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V, i = 1,2,\dots,n$.
(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le min[\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) \le max[\gamma_1(v_i), \gamma_1(vj)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$.

Definition 2.2. An IFG $H = (V', E')$ is said to be an IF subgraph ($IFSG$) of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. That is, $\mu'_{1i} \le \mu_{1i}$; $\gamma'_{1i} \ge \gamma_{1i}$ and $\mu'_{2i} \le \mu_{2i}$; $\gamma'_{2i} \ge \gamma_{2i}$, for every $i, j = 1,2,\dots,n$.

Definition 2.3. Let $G = (V, E)$ be an IFG. Then the cardinality of G is defined to be $|G| = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2} + \sum_{(v_i, v_j) \in E} \frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)}{2}\right|$

Definition 2.4. Let $G = (V, E)$ be an IFG, then the vertex cardinality of V is defined by
$|V| = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right|$ for all $v_i \in V$.

Definition 2.5. Let $G = (V, E)$ be an IFG, then the edge cardinality of E is defined by $|E| = \left|\sum_{(v_i, v_j) \in E} \frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)}{2}\right|$ for all $(v_i, v_j) \in E$.

Definition 2.6. The number of vertices (the cardinality of V) is called the order of an IFG, $G = (V, E)$ and is denoted by $O(G) = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right|$ for all $v_i \in V$.

Definition 2.7. The number of edges (the cardinality of E) is called the size of an IFG, $G = (V, E)$ and is denoted by $S(G) = \left|\sum_{(v_i, v_j) \in E} \frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)}{2}\right|$ for all $(v_i, v_j) \in E$.

Definition 2.8. The degree of a vertex v in an IFG, $G = (V, E)$ is defined to be sum of the weights of the strong edges incident at v. It is denoted by $d_G(v)$.
The minimum degree of G is $\delta(G) = min\{ d_G(v) \mid v \in V \}$.
The maximum degree of G is $\Delta(G) = max\{ d_G(v) \mid v \in V \}$.

Definition 2.9. Two vertices v_i and v_j are said to be neighbors in IFG, if either one of the following conditions hold (i) $\mu_2(v_i, v_j) > 0,\ \gamma_2(v_i, v_j) > 0$ (or) (ii) $\mu_2(v_i, v_j) = 0,\ \gamma_2(v_i, v_j) > 0$ (or)
(iii) $\mu_2(v_i, v_j) > 0,\ \gamma_2(v_i, v_j) = 0,\quad v_i, v_j \in V$.

Definition 2.10. A path in an IFG is a sequence of distinct vertices $v_1, v_2, \dots, v_n$ such that either one of the following conditions is satisfied:
(i) $\mu_2(v_i, v_j) > 0,\ \gamma_2(v_i, v_j) > 0$, for some i and j (or) (ii) $\mu_2(v_i, v_j) = 0,\ \gamma_2(v_i, v_j) > 0$, for some i and j (or) (iii) $\mu_2(v_i, v_j) > 0,\ \gamma_2(v_i, v_j) = 0$, for some i and j.
The length of the path $P = v_1 v_2 \dots v_{n+1}$ is n.

Definition 2.11. If v_i, v_j are vertices in $G = (V, E)$ and if they are connected by means of a path then the strength of that path is defined as $(min\, \mu_{2ij}, max\, \gamma_{2ij})$ for all i, j where $min\, \mu_{2ij}$ is the μ–strength of the weakest arc and $max\, \gamma_{2ij}$ is the $\gamma-$ strength of the strongest arc.

Definition 2.12. If $v_i, v_j \in V \subseteq G$, the μ– strength of connectedness between v_i and v_j is $\mu_2^\infty(v_i, v_j) = sup\{\mu_2^k(v_i, v_j) \mid k = 1, 2, \dots, n\}$ and γ-strength of connectedness between v_i and v_j is $\gamma_2^\infty(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j)$ such that $k = 1, 2, \dots, n\}$. If u, v are connected by means of paths of length k then $\mu_2^k(u, v)$ is defined assup$\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \mu_2(v_2, v_3) \dots \wedge \mu_2(v_{k-1}, v)$ such that $(u, v_1, v_2 \dots v_{k-1}, v \in V)\}$ and $\gamma_2^k(u, v)$ is defined as $inf\,\{\gamma_2(u, v_1) \wedge \gamma_2(v_1, v_2) \wedge \gamma_2(v_2, v_3) \dots \wedge \gamma_2(v_{k-1}, v)$ such that $(u, v_1, v_2 \dots v_{k-1}, v \in V)\}$

Definition 2.13. An IFG, $G = (V, E)$ is said to be complete IFG, if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = max(\gamma_{1i}, \gamma_{1j})$ for every $v_i, v_j \in V$.

Definition 2.14. The complement of an IFG, $G = (V, E)$ is an IFG $\bar{G} = (\bar{V}, \bar{E})$ where
(i) $\bar{V} = V$
(ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$ for all $i = 1, 2, \dots, n$.
(iii) $\overline{\mu_{2ij}} = min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ and $\overline{\gamma_{2ij}} = max(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij}$ for all $i, j = 1, 2, \dots, n$.

Definition 2.15. An IFG, $G = (V, E)$ is said to bipartite, if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that
(i) $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$, if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$,
(ii) $\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) > 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j (or)
$\mu_2(v_i, v_j) = 0, \gamma_2(v_i, v_j) > 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j (or)
$\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) = 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j.

Definition 2.16. A bipartite IFG, $G = (V, E)$ is said to be complete, if $\mu_2(v_i, v_j) = min(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_2(v_i, v_j) = max(\gamma_1(v_i), \gamma_1(v_j))$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by K_{V_1, V_2}.

Definition 2.17. A vertex $u \in V$ of an IFG $G = (V, E)$ is said to be an isolated vertex if
$\mu_2(u, v) = 0$ and $\gamma_2(u, v) = 0$ for all $u, v \in V$.

Definition 2.18. An arc (u, v) is said to be a strong arc, if $\mu_2(u, v) \geq \mu_2^\infty(u, v)$ and $\gamma_2(u, v) \geq \gamma_2^\infty(u, v)$.

Definition 2.19. Let $G = (V, E)$ be an IFG and let $u, v \in V$, we say that u dominates v in G if there exists a strong arc between them.

Note: An isolated vertex does not dominate any other vertex.

Definition 2.20. A subset S of V is called a dominating set in IFG, if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

Definition 2.21. A dominating set S of an IFG is said to be minimal dominating set, if no proper subset of S is a dominating set.

Definition 2.22. Minimum cardinality among all minimal dominating set is called lower domination number of G and is denoted by $d(G)$. Maximum cardinality among all minimal dominating set is called upper domination number of G

and is denoted by $D(G)$.

Definition 2.23. Let $G = (V, E)$ be an IFG. A set $S \subseteq V$ is said to be a complementary nil domination set (or simply cnd-set) of an $IFG\ G$, if S is a dominating set and its complement $V - S$ is not a dominating set. The minimum scalar cardinality over all cnd-set is called a complimentary nil domination number and is denoted by the symbol $\gamma_{cnd}(G)$, the corresponding minimum cnd-set is denote by γ_{cnd}-set.

Definition 2.24. A dominating set D of an intuitionistic fuzzy graph $G = (V, E)$ is a non- split dominating set, if the induced intuitionistic fuzzy sub graph $G(V - D)$ is connected. The non- split domination number $\gamma_{ns}(G)$ of intuitionistic fuzzy graph G is the minimum cardinality of all non-split domination set.

Definition 2.25. A dominating set D of a intuitionistic fuzzy graph $G = (V, E)$ in a split dominating set if the induced fuzzy sub graph $G(V - D)$ is disconnected. The minimum fuzzy cardinality of a split dominating set is called a split domination number and is denoted by $\gamma_s(G)$.

3. MAIN RESULTS:

NOTATIONS: $d(G)$ - minimum domination number, $D(G)$- maximum domination number, $\gamma_{ns}(G)$- minimum non-split domination number, $\Gamma_{ns}(G)$- maximum non-split domination number , $\gamma_s(G)$- minimum split domination number, $\Gamma_s(G)$- maximum split domination number, $\gamma_{cnd}(G)$- minimum complementary nil domination number, $\Gamma_{cnd}(G)$- maximum complementary nil domination number

Theorem 3.1. The edges incident with a vertex $u \in V(G)$ in an $IFG, G = (V, E)$ are not strong edges then u must lie minimal dominating set.

Proof : Let $G = (V, E)$ be an IFG. Let $u \in V(G)$ the edges incident with u are not strong, then for any vertex $v \in V(G)$ we cannot reach to u. Therefore u is not dominated by any other vertices in V. By definition of minimal dominating set, the undominated vertex lie in the minimal dominating set. Therefore u must lie in the minimal dominating set □

Example 3.1

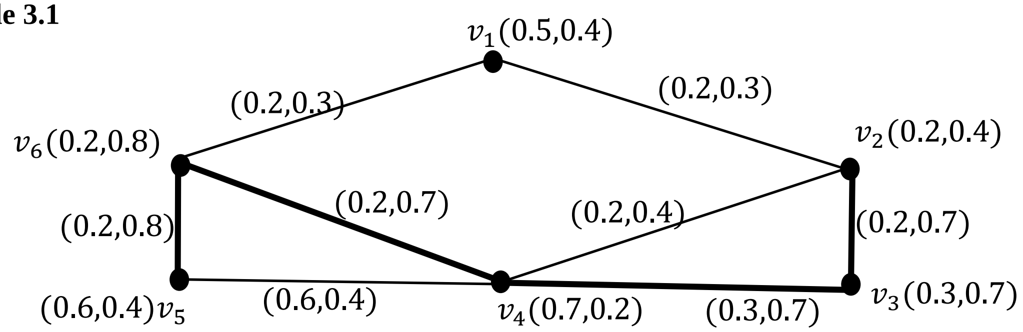

Figure 3.1

In the above figure 3.1 the strong edges are $e_{23}, e_{34}, e_{46}, e_{56}$. The minimal dominating sets are $D_1 = \{v_1, v_3, v_6\}$ and $D_2 = \{v_1, v_2, v_6\}$. The incident edges of v_1 are not strong. Therefore v_1 lie in minimal dominating set.

Theorem 3.2. The edges of an IFG are not strong then the dominating set is V and it is unique.

Proof : Let $G = (V, E)$ be an IFG. Suppose that all the edges are not strong then by theorem 3.1, for every $u \in V$ there is no dominated vertex $v \in V$. Therefore all the vertices of G lie in the dominating set. Hence the dominating set is V . Suppose G has two dominating set $V_1$and V_2. We find that either number of vertices of $V_1$is greater than number of vertices of V_2 or number of vertices of $V_2$is greater than number of vertices of V_1. Which is not possible because either V_1or V_2 contains all the vertices of G. Therefore $V_1 = V_2$. Hence the domin□ing set is unique.

Corollary 3.3. The edges of an IFG are not strong then $d(G) = D(G) = O(G)$.

Proof: Let $G = (V, E)$ be an IFG. The edges of G are not strong then by theorem 3.2 the only dominating set is V. By the definition of domination number $d(G) = D(G) = |V|$--------(1)

We know that $|V| = O(G)$------------(2).

Combining these two equations $d(G) = D(G) = O(G)$.

Theorem 3.4. In any $IFG, G = (V, E)$, the non-split domination and split domination are avoid one another.

Proof : Let $G = (V, E)$ be an IFG and D be the minimal dominating set of G. Then we find that induced IFG, $G[V - D]$ is either connected or disconnected. Suppose $G[V - D]$ is connected then by definition of non-split dominating set, G is non-split domination. Otherwise $G[V - D]$ is disconnected then by definition of split dominating set, G is split domination. Therefore either one of the case is occur in G. Hence non-split domination and split domination are avoid one another.

□

Corollary 3.5. The edges of an IFG are not strong then there exists neither non-split domination nor split domination.
Proof : Let $G = (V, E)$ be an IFG and let D is the minimal dominating set of G. Since the edges are not strong then by theorem 3.2 the only dominating set is V(that is $D = V$). By the definition of non-split and split domination $V - D$ is an empty set. Therefore G is neither non-split domination nor split domination.
Example 3.2. □

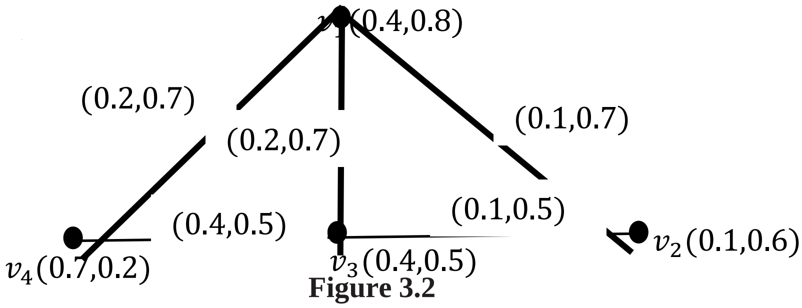

Figure 3.2

In the above figure 3.2 the strong edges are e_{12}, e_{13}, e_{14}. Let $D = \{v_1\}$ then $V - D$=$\{v_2, v_3, v_4\}$. The induced $IFG, G[V - D]$ is connected.

Example 3.3.

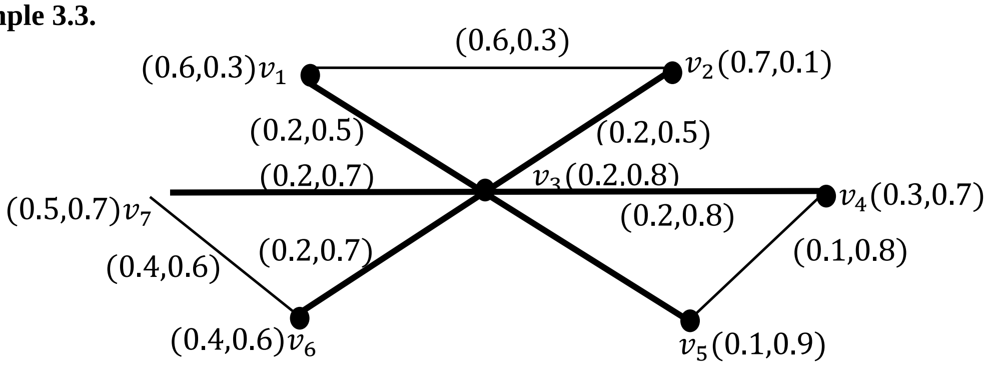

Figure 3.3

In the above figure 3.3 the strong edges are $e_{13}, e_{23}, e_{34}, e_{35}, e_{36}, e_{37}$. Let $D = \{v_3\}$. Then $V - D$=$\{v_2, v_3, v_4, v_5, v_6, v_7\}$. The induced $IFG, G[V - D]$ is disconnected.
Therorem 3.6. In any $IFG\ G = (V, E)$, if there exists only one minimal dominating set then any one of the following condition holds

(i) $d(G) = D(G) = \gamma_{ns}(G) = \Gamma_{ns}(G)$ (or)
(ii) $d(G) = D(G) = \gamma_s(G) = \Gamma_s(G)$

Proof : Let $G = (V, E)$ be an IFG and let D be the only one minimal dominating set of G.
Case(i): $G[V - D]$ is connected

Since D is the only one dominating set then by definition of domination number $d(G) = D(G) = |D|$. We find an induced $IFG\ G[V - D]$ is connected then by definition of non-split domination number $\gamma_{ns}(G) = \Gamma_{ns}(G) = |D|$. Therefore $d(G) = D(G) = \gamma_{ns}(G) = \Gamma_{ns}(G)$
Case(ii): $(V - D)$ is disconnected

Since D is the only one dominating set then by definition of domination number $d(G) = D(G) = |D|$. We find an induced $IFG\ G[V - D]$ is disconnected then by definition of split domination number $\gamma_s(G) = \Gamma_s(G) = |D|$. Therefore $d(G) = D(G) = \gamma_s(G) = \Gamma_s(G)$ □
Example 3.4. In the above figure 3.2. the strong edges are e_{12}, e_{13}, e_{14}. Let $D = \{v_1\}$. Then $V - D$=$\{v_2, v_3, v_4\}$. The induced $IFG, G[V - D]$ is connected. Therefore $d(G) = D(G) = \gamma_{ns}(G) = \Gamma_{ns}(G) = 0.3$. In the above figure 3.3 the strong edges are $e_{13}, e_{23}, e_{34}, e_{35}, e_{36}, e_{37}$. Let $D = \{v_3\}$. Then $V - D$=$\{v_2, v_3, v_4, v_5, v_6, v_7\}$. The induced $IFG, G[V - D]$ is disconnected. Therefore $d(G) = D(G) = \gamma_s(G) = \Gamma_s(G) = 0.2$

Theorem 3.7. If more than one minimal dominating exists of an IFG G=(V,E) then the following conditions holds

(i) $d(G) = \gamma_{ns}(G) \leq D(G) = \Gamma_{ns}(G)$ (or))
(ii) $d(G) = \gamma_s(G) \leq D(G) = \Gamma_s(G)$

Proof:

Case(i): $G(V-D)$ is connected

Let $G=(V,E)$ be an IFG . If G has more than one minimal dominating set and let it be denoted as $D_i(i=1,2,\dots n)$. We find that $d(G)=\min\{D_i(i=1,2,\dots n)\}$. Since $G[V-D]$ is connected then by definition of non-split domination numb $\gamma_{ns}(G)=\min\{D_i(i=1,2,\dots n)\}$.Therefore $d(G)=\gamma_{ns}(G)=\min\{D_i(i=1,2,\dots n)\}$. Similarly we find that $D(G)=\Gamma_{ns}(G)=\max\{D_i(i=1,2,\dots n)$ Therefore $d(G)=\gamma_{ns}(G)=\min\{D_i(i=1,2,\dots n)\}\le\max\{D_i(i=1,2,\dots n)\}=D(G)=\Gamma_{ns}(G)$.

Case(ii): $G(V-D)$ is disconnected

Let $G=(V,E)$ be an IFG . If G has more than one minimal dominating set and let it be denoted as $D_i(i=1,2,\dots n)$. We find that $d(G)=\min\{D_i(i=1,2,\dots n)\}$. Since $G[V-D]$ is disconnected then by definition of split domination number $\gamma_s(G)=\min\{D_i(i=1,2,\dots n)\}$. Therefore $d(G)=\gamma_s(G)=\min\{D_i(i=1,2,\dots n)\}$. Similarly we find that $D(G)=\Gamma_s(G)=\max\{D_i(i=1,2,\dots n)$. Therefore $d(G)=\gamma_{ns}(G)=\min\{D_i(i=1,2,\dots n)\}\le\max\{D_i(i=1,2,\dots n)\}=D(G)=\Gamma_{ns}(G)$. □

Example 3.6.

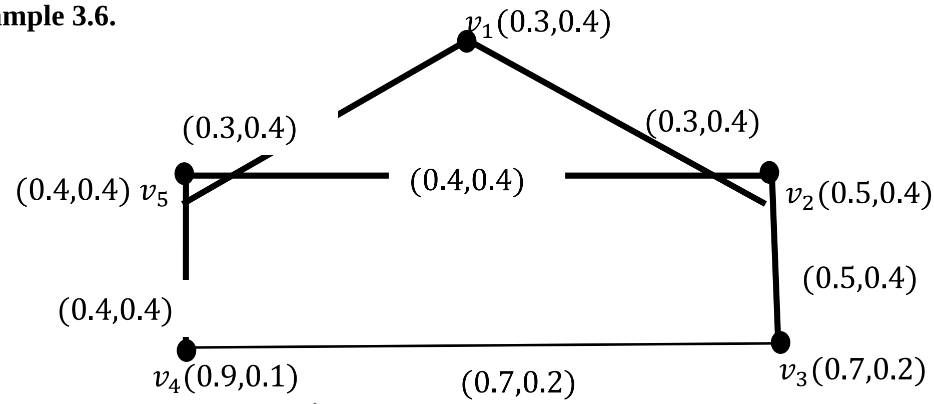

Figure5.4

In figure 5.4 the strong edges are $e_{12},e_{23},e_{45},e_{25},e_{15}$. Here $D_1=\{v_2,v_4\}$, $D_2=\{v_3,v_5\}$, $D_3=\{v_2,v_5\}$. $V-D_1$=$\{v_1,v_3,v_5\}$, $V-D_2$=$\{v_1,v_2,v_4\}$ and $V-D_3$=$\{v_1,v_3,v_4\}$are disconnected. $d(G)=\gamma_s(G)$ =1.05 and $D(G)=\Gamma_s(G)=1.35$.Therefore $d(G)=\gamma_s(G)\le D(G)=\Gamma_s(G)$

Theorem 3.8 : For any IFG G=(V,E), if the minimal complementary nil dominating set is similar to the minimal dominating set then the following conditions holds

(i) $d(G)=\gamma_{ns}(G)=\gamma_{cnd}(G)\le D(G)=\Gamma_{ns}(G)$=$\Gamma_{cnd}(G)$ (or)

(ii) $d(G)=\gamma_s(G)=\gamma_{cnd}(G)\le D(G)=\Gamma_s(G)$=$\Gamma_{cnd}(G)$

Proof:

Case(i): $G[V-D]$ is connected

Let $G=(V,E)$ be an IFG and let D is the minimal dominating set. Since D is the minimal complementary nil dominating set then we find that the induced $IFG, G(V-D)$ is connected. By definition $d(G)=\gamma_{ns}(G)=\gamma_{cnd}(G)=|D|$. Similarly we prove that $D(G)=\Gamma_{ns}(G)$=$\Gamma_{cnd}(G$. Since more than one minimal dominating set exists by theorem(3.7) $d(G)=\gamma_{ns}(G)=\gamma_{cnd}(G)\le D(G)=\Gamma_{ns}(G)$=$\Gamma_{cnd}(G)$

Case(ii): $G(V-D)$ is disconnected

Let $G=(V,E)$ be an IFG and let D is the minimal dominating set. Since D is the minimal complementary nil dominating set then we find that the induced $IFG, G(V-D)$ is connecte. By definition $d(G)=\gamma_s(G)=\gamma_{cnd}(G)$. Similarly we prove that $D(G)=\Gamma_s(G)$=$\Gamma_{cnd}(G$. Since more than one minimal dominating set exists by theorem(3.7) $d(G)=\gamma_s(G)=\gamma_{cnd}(G)\le D(G)=\Gamma_s(G)$=$\Gamma_{cnd}(G)$. □

Example 3.6.: Consider the figure 5.1 the strong edges are $e_{23},e_{34},e_{46},e_{56}$. The minimal dominating sets are $D_1=\{v_1,v_3,v_6\}$ and $D_2=\{v_1,v_2,v_6\}$.

$d(G)=\gamma_s(G)=\gamma_{cnd}(G)=1.05$ and $D(G)=\Gamma_s(G)$=$\Gamma_{cnd}(G)=1.15$.

Therefore $d(G)=\gamma_s(G)=\gamma_{cnd}(G)\le D(G)=\Gamma_s(G)$=$\Gamma_{cnd}(G)$

Theorem 3.9: For any IFG G=(V,E), the complementary nil dominating set is different from dominating set then the following condition holds

(i) $d(G)=\gamma_{ns}(G)\le D(G)=\Gamma_{ns}(G)\le\gamma_{cnd}(G)\le\Gamma_{cnd}(G)$ (or)

(ii) $d(G)=\gamma_{ns}(G)\le D(G)=\Gamma_{ns}(G)\le\gamma_{cnd}(G)\le\Gamma_{cnd}(G)$

Proof: Case(i): $G(V-D)$ is connected

Let $G = (V, E)$ be an IFG and let D is the minimal dominating set of G. By theorem(3.7) $d(G) = \gamma_{ns}(G) \leq D(G) = \Gamma_{ns}(G)$.Since minimal complementary nil dominating set is different from minimal dominating set then by definition minimum domination number is less than or equal to minimum complementary nil domination number and again this is less than or equal to maximum complementary nil domination number. therefore

$$d(G) = \gamma_{ns}(G) \leq D(G) = \Gamma_{ns}(G) \leq \gamma_{cnd}(G) \leq \Gamma_{cnd}(G)$$

Case(ii): $G(V - D)$ is disconnected

Let $G = (V, E)$ be an $IFG. D$ is the minimal dominating set. By theorem(3.8) $d(G) = \gamma_s(G) \leq D(G) = \Gamma_s(G)$ Since minimal complementary nil dominating set is different as minimal dominating set then by definition minimum domination number is less than or equal to minimum complementary nil domination number and again this is less than or equal to maximum complementary nil domination number. Therefore $d(G) = \gamma_s(G) \leq D(G) = \Gamma_s(G) \leq \gamma_{cnd}(G) \leq \Gamma_{cnd}(G)$

Example 3.6.: In figure 3.4 the strong edges are $e_{12}, e_{23}, e_{45}, e_{25}, e_{15}$. Here $D_1 = \{v_2, v_4\}$, $D_2 = \{v_3, v_5\}$, $D_3 = \{v_2, v_5\}$. $V - D_1$=$\{v_1, v_3, v_5\}$, $V - D_2$=$\{v_1, v_2, v_4\}$ and $V - D_3$=$\{v_1, v_3, v_4\}$are disconnected. $d(G) = \gamma_s(G)$ =1.05 and $D(G) = \Gamma_s(G) = 1.35$. The minimal complementary nil dominating sets are $S_1 = \{v_1, v_2, v_3\}$, $S_2 = \{v_2, v_3, v_4\}$, $S_3 = \{v_3, v_4, v_5\}$, $S_4 = \{v_1, v_4, v_5\}$, $S_5 = \{v_2, v_4, v_5\}$, $S_6 = \{v_1, v_3, v_5\}$,and $S_7 = \{v_1, v_2, v_5\}$. $\gamma_{cnd}(G) = 1.5$ and $\Gamma_{cnd}(G) = 2.2$. Therefore $d(G) = \gamma_s(G) \leq D(G) = \Gamma_s(G) \leq \gamma_{cnd}(G) \leq \Gamma_{cnd}(G)$

4. CONCLUSION:

Some relationship among the domination parameter in intuitionistic fuzzy graph is introduced. Some properties of dominating set, split and non-split domination in IFG are discussed.

REFERENCES:

1. K. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Physica- Verlag, New York (1999).
2. J.A. Bondy and U. S. R. Murthy, Graph Theory with Applications, American Elesevier Publishing Co., New York (1976).
3. G.Chartrand, L.Lesniak, Graphs and Digraphs, Chapman and Hall/CRC, New York, 1996.
4. E. J. Cockayne and S. T. Hedetnieme, Towards a Theory of Domination in Graphs, Networks 7 (1977), 247-261.
5. R.Jahir Hussain and S.Yahaya Mohamed, Complementary Nil Domination in Intuitionistic fuzzy Graph, International mathematics Forum, Vol.9, 2014, No.23, 1101-1108
6. Nagoor Gani, A. and Shajitha Begum, S., Degree, Order and Size in Intuitionistic Fuzzy Graphs, International Journal of Algorithms Computing and Mathematics, Vol. 3, No.3, p.11-16, 2010.
7. A.Nagoor Gani and S.Anupriya, Non-Split Domination on Intuitionistic Fuzzy Graphs, International Journal of Fuzzy Mathematical Archieve, Vol.7, No.1, 2015, 51-62.
8. A.Nagoor Gani and S.Anupriya, Split Domination on Intuitionistic Fuzzy Graphs, Advances in computational Mathematics and its Applications(ACMA) Vol.2, No.2, 2012, ISSN 2167-6356.
9. Parvathi, R., Karunambigai, M.G., and Atanassov, K., Operations on Intuitionistic Fuzzy Graphs, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 1396-1401, August 2009.
10. Parvathi, R., Karunambigai, M.G., Intuitionistic Fuzzy Graphs, Computational Intelligence, Theory & Applications, International Conference in Germany, Sept 18-20, 2006.
11. Parvathi, R., and Tamizhendhi, G., Domination in intuitionistic fuzzy graphs, Fourteenth Int. Conf. on IFSs, Sofia, NIFS Vol. 16, 2, 39-49, 15-16 May 2010.